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Dynamics of a Simple Rotor System

Mili J.Hota[1] and D.P. Vakharia[2]

[1,2]Sardar Vallabhbhai National Institute of Technology, Surat
E-mail: [1]milihota@yahoo.com, [2]vakharia@med.svnit.ac.in

Abstract: *This paper describes how to develop a mathematical model of a rotordynamical system for a reader either is a beginner or has a background in dynamical systems. The theory is described in order to conceive the main idea how to develop dynamical models of rotors. The problem is based on FEM and described in detail. The comparison of results is done with help of MATLAB code using the ROTORDYNAMIC TOOLBOX and Ansys modal analysis. The purpose of this paper, as mentioned above, is to identify why calculations are not the same or sometimes not even close at all to the real values. Thus, the main goal is, to track down what more should be taken into consideration in the computation (like bearings, support and so forth).*

1. INTRODUCTION

Rotating machinery are found in many applications from small pumps and compressors to large gas turbine generators and aircraft engines. These rotating machines have many different components each of which has to be designed to withstand different operating conditions. In a jet engine application, static structures have to maintain the stiffness and rigidity of the machine as they operate in a variety of environments and maneuvers. However, a typical gas turbine engine has at least one rotating shaft that contains bladed disks which are responsible for either compressing air or extracting energy from a combination of air and fuel exhaust. The rotor system produces the power or thrust for these systems. The gas turbine engine rotor section, with a large multi-stage compressor connected by a shaft to a multistage turbine is the best example.

The study of the dynamic response of the rotating components is called rotordynamics. Rotor systems can vibrate longitudinally, which is an extension or compression of a rotor. They can also vibrate torsionally, which is an oscillation of the twist of a rotor. The most important and difficult vibrations to understand are lateral vibrations of a rotor system. Unlike a simple beam, the rotation of the rotor system and gyroscopic moments create a lateral motion that can be visualized as an orbit. Understanding the lateral vibrations of a rotor system is extremely important for stable and safe operation. It is important to understand and avoid critical speeds during operation. It is also be important in jet engine applications to assure proper deflections and corresponding clearances are held.

2. ROTOR DYNAMIC MODELING AND ANALYSIS

During the last 50 years, engineers have developed several new techniques and solved many problems that industry is facing in dynamics of rotating machines. However the demands and reliability on machines that can operate in hard conditions are expected to increase.

Some remarkable references are:

- Active vibration control of rotor by Kari Tammi with the use of special accessory to develop magnetic field round the rotor [1].
- Response of a warped flexible rotor with a uid bearing by Jim Meagher, Ci Wu and Chris Lencioni [2].
- Investigation of vibration of a rotor system supported by absolutely rigid bearings with a shaft containing a notch by Petr Ferfecki, Jan Ondrouch and Tomas Lukas [3].
- The main purpose was to study a rotor system with a shaft weakened by a notch. Determination of oil whip phenomena by Agnes Muszynska [4].
- Experiment design for simulating di_erent faults in a rotor kit by Enayet Halim [5]. A magnetorheological uid damper for rotor applications by P. Forte, M. Paterno, and E. Rustighi [6].

3. FINITE ELEMENT EQUATIONS

The rotor-bearing system is modelled using finite element models .A global equation of motion, Equation (1), is obtained from the finite element matrices, where [M] represents the global shaft translational inertia matrix, [N] represents the global rotatory inertia matrix, [K] the shaft and bearing stiffness matrix and [C] is the generalized shaft and bearing damping matrix, in which the shaft gyroscopic effects are included. The bearings stiffness [Km] and damping [Cm] coefficients are included into the system matrices, in order to represent the fluid film resistance to the rotor displacement and to velocity, respectively. The rotor-bearing system equation is rewritten on state form to compute the complex eigenvalues. The complex eigenvalues associated with the system are separated to get the natural frequencies and information on the stability of the rotor-bearing system.

$$[M+N]\{\ddot{U}\}+[C]\{\dot{U}\}+[K]\{U\}=\{R\} \qquad .....(1)$$

where [M] represents the global shaft translational inertia matrix, [N] represents the global rotatory inertia matrix, [K] the shaft and bearing stiffness matrix and [C] is the generalized shaft and bearing damping matrix, which is expressed as [C]=[C1]- Ω[G] in which [G] is the shaft gyroscopic effect matrix. The matrix [C1] represents the bearing damping. The external excitation force is represented by the vector {R} in Equation (1). For the eigenvalue problem analysis, this vector id null {R}={0}.

$$[U]=\begin{bmatrix} y_i \\ x_i \\ \varphi_i \\ \theta_i \end{bmatrix} \qquad (2)$$

y_i=horizontal displacement

x_i=vertical displacement

φ_i=rotation around the y axis

θ_i=rotation around the x axis

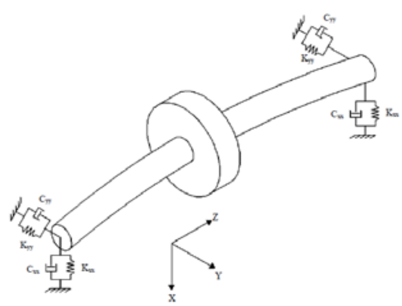

Fig. 1: Flexible shaft supported on fluid film journal bearings.

Bearing Modelling

The journal bearing finite element model is developed based on the classical Reynolds equation for oil-lubricated plain cylindrical journal bearings [1]. For the coordinates (X, Z), this equation is given by.

$$\frac{\partial}{\partial X}\left(\frac{h3}{12\mu}\frac{\partial P}{\partial X}\right)+\frac{\partial}{\partial Z}\left(\frac{h3}{12\mu}\frac{\partial P}{\partial Z}\right)=\frac{\Omega R}{2}\frac{\partial P}{\partial Z}+\frac{\partial h}{\partial t} \qquad (3)$$

The journal rotational speed is denoted by Ω. Journal eccentricities on the vertical and horizontal directions are expressed as eX and eY, respectively. The eccentricity ratio is defined as ε = e/c, where e2 = ex2 + ey2. The circumferential coordinate X = R· θ and R is the bearing radius. Fluid viscosity is given by μ, P represents the hydrodynamic pressure and h is the fluid film thickness. A linearized perturbation procedure is used in conjunction with Equation (3) to render the zeroth- and first-order lubrication equations [19]. These equations allow the computation of the bearing reaction forces and eight dynamic force coefficients. For brevity, these equations and the validation of the finite element procedure for the bearing dynamic coefficients are omitted in this work. The dynamic force coefficients are represented in matrix form by the stiffness [Km] and the damping [Cm] matrices as in Equation (4), given by [4]. They stand for the fluid film resistance to the rotor displacement and velocity, respectively.

$$[K_m]=\begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix};[C_m]=\begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix} \qquad (4)$$

Fig. 2 depicts the cross-section of a journal bearing and its linearized stiffness and damping coefficients along the X-axis and Y-axis.

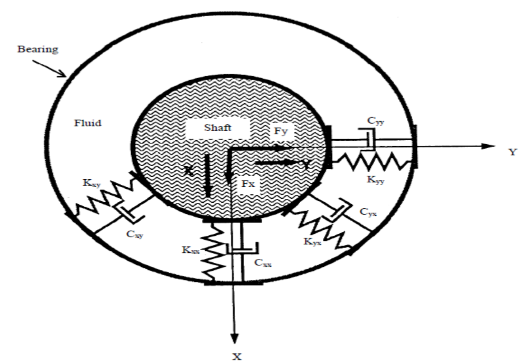

Fig. 2: Linearized stiffness and damping coefficients of the journal bearing.

4. EIGENVALUE PROBLEM

The vibration analysis of rotor-bearing systems can be carried out through computational procedures developed specially to predict the dynamic response and stability analysis of rotating shafts supported by fluid film bearings. At the preliminary design and commissioning stages of industrial turbo machinery, those procedures can bring important insights on the rotating system dynamic behavior. The first step in the dynamic analysis consists of obtaining the system natural frequencies under several operating conditions. The free vibration problem associated with linear systems of differential equations leads naturally to the eigenvalue problem [4]. For damped gyroscopic systems, the complex eigenvalues and eigenvectors provide very useful data about the mode shapes and stability of rotating systems. The eigenvalue problem associated with Equation (1) can be reduced to a standard form, following a procedure similar to that presented by [2]. A second order state vector {X}, defined in the following form, is used to rewrite the governing equation on state variables:

$$\{X\}=\left[\{\dot{U}\}^{T}\ \{U\}^{T}\right] \tag{5}$$

The free vibration problem associated with Equation (1) can be rewritten as follows where [I] is the identity matrix, with the same dimension as that of [M], [N], [C] and [K]. The solution of Equation (6) has the form

$$\left[M^{*}\right]\{\dot{X}\}+[C]\{X\}=\{0\}$$

$$\left[M^{*}\right]=\begin{bmatrix}[M]+[N] & 0\\ 0 & [I]\end{bmatrix}$$

$$[C]=\begin{bmatrix}[C] & [K]\\ -[I] & 0\end{bmatrix} \tag{6}$$

$$[A^{*}]\{X_0\}=S\{X_0\} \tag{7}$$

and the associated eigenvalue problem can be stated as

$$s[M^{*}]\{X\}+[C]\{X_0\}=\{0\} \tag{8}$$

where, provided [M*] is non-singular,

$$[A^{*}]=-[M^{*}]^{-1}[C]=\begin{bmatrix}-([M]+[N])^{-1}[C] & -([M]+[N])^{-1}[K]\\ [I] & 0\end{bmatrix} \tag{10}$$

5. NUMERICAL RESULTS

A simple two disk rotor was modeled to demonstrate a dynamic analysis of a rotor system. Rotor models typically consist of a shaft, an orientation of several disks, and bearings to support the structure. In this analysis the bearings were assumed to be isotropic with a stiffness of 1 MN/m at either end of the rotor. The rotor modeled has a 1.5 meter long shaft with a .05 meter diameter. The two disks are modeled equally spaced from either end at .5 meters and 1.0 meters along the shaft. The first disk had a diameter of .28 meters and a thickness of .07 meters, while the second disk had a diameter of .35 meters and also a thickness of .07 meters. The rotor is assumed to be made from steel with E=211 GN/m^2 and ρ=7810 kg/m3. The analysis was run from 0 to 4000 RPM.

Table 1: System parameters adopted for the analysis.

Parameter	Description	Value	Unit
L	shaft length	1.5	M
d	shaft diameter	0.05	M
E	shaft Young module	211	GN/m2
ρ	shaft specific mass	7810	kg/m³
kb	bearings direct stiffness coefficient	1	MN/m
cb	bearings direct damping coefficient	------	N·s/m
No.of disks	Equally spaced at 0.5m and1 m	Two	
D1	Diameter of first disk	0.28	m
T1	Thickness of first disk	0.07	m
D2	Diameter of second disk	0.35	m
T2	Thickness of second disk	0.07	m

6. MATLAB ANALYSIS

This rotor was model using two different methodologies. The first model was produced using software provided with Dynamics of Rotating Machines (Friswell et al). This software produces a simple Finite Element Model that is solved using Matlab. Fig. 3 is a representation of the Friswell model in Matlab. This software is a set of scripts written in MATLAB to accompany the above book. The primary purpose of the software is to illustrate features of rotating machines described in particular the lateral motion based on shaft-line models.

Of course the software can be used to analyze other rotating machines and for research purposes, although there are no guarantees implied with the software and it is the user's responsibility to confirm the suitability and accuracy of the results. The software is open source.

At a basic level the software consists of three aspects:

- Defining the model, forcing and operating conditions;
- Analyzing the system and generating the results; and
- Graphical means for interpreting the model and results.

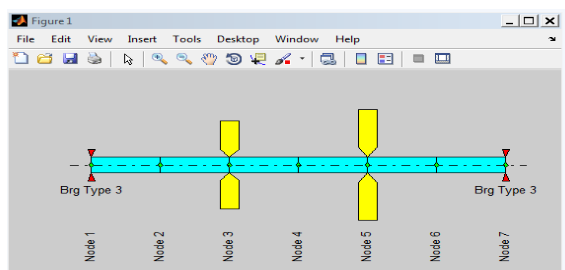

Fig. 3: MATLAB model

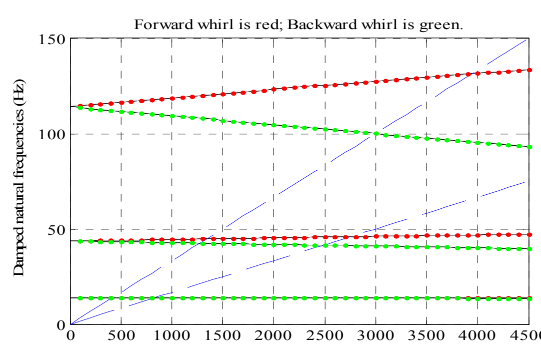

Fig. 4: MATLAB Campbell Diagram

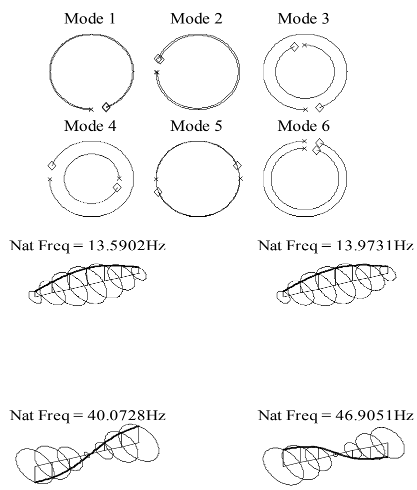

Fig. 5: Mode shapes and Rotor Orbits

7. ANSYS ANALYSIS

The rotor was also modeled using ANSYS finite element software. The model has 16 nodes with 15 Beam188 elements equal spaced every 0.1 meters along the shaft. The disks were modeled as Mass21 elements at node 6 and node 11, and the moments of inertia and masses were calculated for each disk and input as real constants for each element. The bearings are modeled at the beginning and end of the shaft as Combi214 elements. The stiffness and damping constants in the lateral directions are input as real constants.

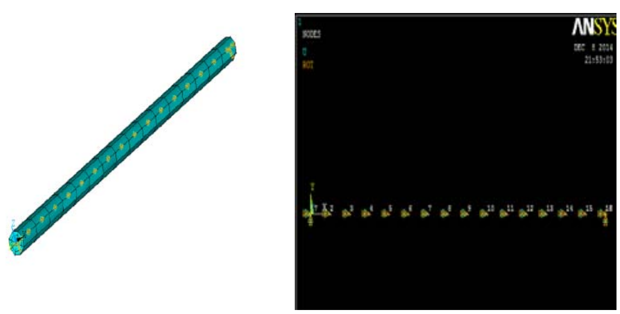

Fig. 6: Ansys model

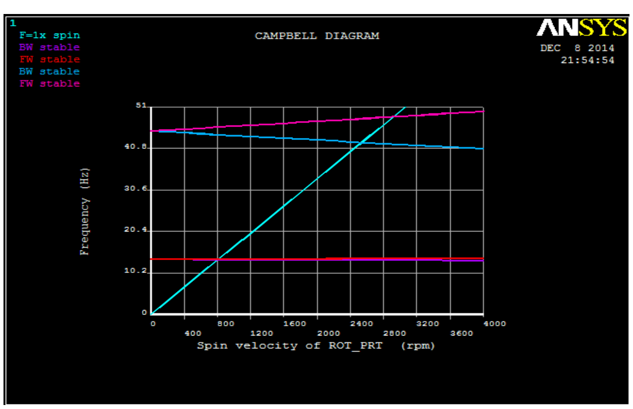

Fig. 7: Ansys Campbell Diagram

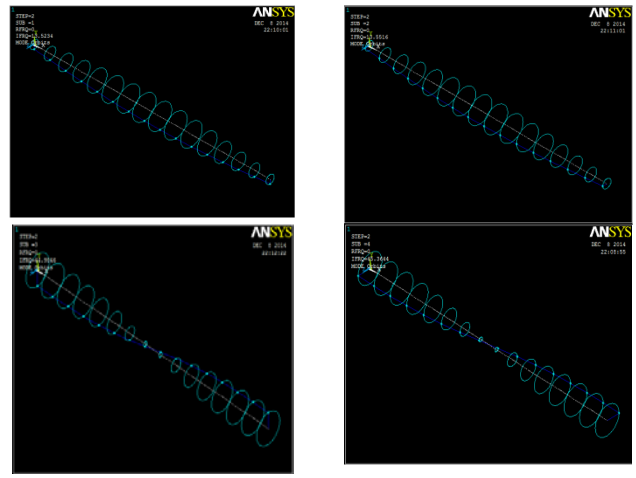

Fig. 8: Mode shapes and Rotor Orbits on Ansys

Each node is constrained in the longitudinal (x) direction, while the bearing nodes on either end are constrained in all degrees of freedom. The model was run rotating about the x axis with the Coriolis Effect on from 0 to 4000 RPM. Fig. 6 shows the ANSYS model, which was solved using the Block-Lanczos mode extraction method.

8. RESULTS AND DISCUSSION

Both models produce frequencies, mode shapes and orbits for the first four modes of the system. Fig. 4 shows the Campbell diagram for the rotor model from 0 to 4000 RPM from the Matlab model.

The Campbell diagram shows the backward whirling modes in green and the forward whirling modes in red. One can also see that the forward whirling modes increase in frequency with increasing rotor speed, while backward whirling modes decrease frequency with increasing rotor speed. Fig. s 5 show the mode shapes and orbits respectively as well as the frequencies at 4000 RPM. The mode shapes in Fig. 5 show the relative shape, which is circular, but the amplitudes are arbitrary. The modes are all circular as a result of the isotropic bearings and would be elliptical if the bearing were not isotropic. One may also notice that the odd modes are backwards whirling and the even modes are forward whirling and that the modes 1 & 2 and modes 3 & 4 are pairs of similar modes.

Fig. 7 shows the Campbell Diagram produced from the ANSYS model using the PLCAMP command from the rotor dynamics menu.

This Campbell diagram shows the backward whirling modes in purple and blue, and the forward whirling modes in pink and red. The diagram is consistent with Matlab with forward whirling modes increasing in frequency with increasing rotor speed, while backward whirling modes decrease frequency with increasing rotor speed. Fig. s 8 shows the mode shapes and orbits using the PLORB command in for the first 4 modes in ANSYS.

Modes 1 and 2 are first lateral modes with circular orbits, one forwards one backwards, and are the same as those produced by Matlab. Modes 3 and 4 are second lateral modes also with circular orbits and with the center of the rotor motionless just like the Matlab orbits. Overall, the Campbell diagrams, mode shapes, and orbits of both the Matlab model and the ANSYS beam model are in agreement. Table 2 shows that the first 4 natural frequencies are in good agreement and are within 1-2% of each other.

Table 2; Model frequency comparison.

	Matlab-Friswell (Hz)	ANSYS (Hz)	Delta
Mode 1	13.5	13.3	1.50%
Mode 2	13.97	13.8	1.23%
Mode 3	40.1	40.7	-1.47%
Mode 4	46.91	47.9	-2.07%

9. CONCLUSION

The academic Matlab code and commercial FEM software ANSYS are good tools for understanding the dynamics of rotating systems. Although the Matlab software was sufficient for this simple system, ANSYS would be the tool of choice for modeling more complex systems. ANSYS has a rotor dynamics menu that allows for automatic post processing of Campbell diagrams and orbits. The ability to use the animation features in ANSYS are also really good, especially for understanding the direction and shape of the rotor whirling.

Understanding the dynamics of a rotor system is critical for safe and efficient operation. The dynamics of a rotor system are similar to non-rotating systems, with the increased complexity of gyroscopic moments acting on the system. Both simple and more complex rotor systems can be modeled numerically using the Finite Element Method to understand their frequencies and orbit characteristics.

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